The Extremal Charges Method in Grounding Grid Design

E. Bendito, A. Carmona, A.M. Encinas, M.J. Jiménez

Abstract-- The behaviour of the electric field in earth generated by a fault current derivation to a grounding grid can adequately be modeled by a Coulombian potential that is constant in the grid and its symmetrical. We aim here at approximating, in an efficient way, the charge distribution that produces this potential by using Potential Theory methods. Firstly we describe the extremal charges method that allows to obtain an approximation to the charge distribution as the solution of an optimization problem, specifically a linear programming problem. Secondly, we show the accuracy of the method obtaining the potential and the fundamental parameters of a grid. Then, we describe a grid design optimization methodology. The optimal grid performance is the one that achieves the better agreement between the grid resistance and the touch and step voltages. Both parts are illustrated with graphical results showing the accuracy of this new method of grid analysis and design.

***Index Terms*-- Computer modeling, grid resistance, grounding, measurements, optimization of grounding grids, safety, step potential, touch potential.**

I. Introduction

THE estimation of the grid resistance values and the touch and step voltages, is usually carried out by means of formulas and algorithms that take into account the mutual influence between the grid electrodes. The methodology must reflect the physical phenomenon which shows that the current derived to earth is distributed in the grounding grid in such a way that the potential is constant in it.

The properties of the electrical potential have allowed us to estimate the current distribution in the grid by using the so-called *extremal charges method* developed by some of the authors in [1]. A good approximation to the current distribution, that makes constant the potential on the grounding grid, is performed by a linear programming algorithm.

To evaluate the quality of this approximation, we apply this methodology to several academic grids. First of all we check that the obtained solution provides a nearly constant potential on the grid surface. Then, the computed current distribution can be used to estimate the grid resistance and the potential anywhere. Later, we analyze whether the values of the fundamental parameters can be improved by considering unequally spaced grids and/or peripheral ground rods, since both operations smooth the current distribution. Although it is clear that the incorporation of electrodes or rods to a grid reduces the values of its fundamental parameters, this can also be done by changing the grid performance while keeping the conditions of location and the quantity of material. The results confirm that the use of grids with optimized geometries can supply savings of material while preserving the security in the substations.

In order to choose the grounding grid design that better fits the specific necessities, it is useful to dispose of a nimble but accurate method that allows to discriminate between different grid performances and to know precisely the zones of greater risk. The values of the current distribution are obtained as the solution of a linear system whose coefficients are usually computed by considering the influence between the linear segments into which the grounding electrodes have been broken up. The most usual methods to construct the influence matrix are the Average Potential Method ([2],[3]), the Method of Moments ([4]), the Charge Simulation Method ([5],[6]) and the Boundary Element Method ([7],[8].) If the influence matrix was built in an accurate way and the resolution of the linear system provided a solution greater than zero, any of the above methods would lead to a good approximation of the current distribution. Well now, the influence coefficient matrix obtained by applying any of them, has not any algebraic property guaranteeing that the obtained charge coefficients are positive. As far as we now, the restriction on the charge coefficient of being greater than or equal to zero is not imposed even when the problem is tackled by an optimization method like least squared error method.

It is well-known that the smaller the segments the better approximation, but if the segmentation were refined enough the coefficient of the influence matrix corresponding to nearby elements would produce numerical instabilities, see ([8],[9]).

The extremal charges method solves these difficulties with simplicity. Firstly, the methodology that we use to construct the influence matrix assures that their coefficients are bounded for any geometry and for any electrode segmentation. Results of Potential Theory, in which is based the extremal charges method, allow to obtain, by solving a linear programming problem, the better positive approximation to the solution of the corresponding system.

II. Extremal charges method

In this section we describe a new method to compute the electrical potential. For the sake of simplicity we will assume

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that the soil is homogeneous. Nevertheless, the consideration of an heterogeneous soil, that is often represented as a multilayer model, does not produce any dysfunction in our calculus method and it only give an increase in the number of operations needed to compute the influence matrix. Besides, if we assume that the ground surface is flat, then the method of images can be used and hence the potential will be characterized for being constant on the grounding grid and its symmetrical. Specifically, if D denotes the grounding grid and $\partial(D)$ denotes its boundary, then the current distribution σ verifies for all $x \in \partial(D)$

$$V^{\sigma}(x) = \frac{\rho}{4\pi}\int_{\partial(D)} \sigma(y)\left(\frac{1}{\|x-y\|} + \frac{1}{\|x-y'\|}\right) dS(y) = 1, \qquad (1)$$

where ρ is the soil resistivity, y' is the symmetrized of y and $\|x-y\|$ is the euclidean distance in $\Re^3$. It is well-known that the grid resistance is obtained from σ as

$$R = \frac{\rho}{2\pi}\left(\int_{\partial(D)} \sigma(y)\, dS(y)\right)^{-1}.$$

On the other hand, in the context of Potential Theory, problem (1) is known as *equilibrium problem* for the grounding grid and its symmetrical. Moreover, this equilibrium problem is equivalent to the following optimization problem, (see [10]): *obtain* $\mu^* \in M^1(D)$ *producing*

$$\min_{\mu \in M^1(D)} \max_{x \in \partial(D)} V^{\mu}(x), \qquad (2)$$

where $M^1(D)$ is the set of positive charge distributions in D such that $\int_D \mu(y)\, dV(y) = 1$. In addition, μ^* is concentrated on $\partial(D)$, its potential is constant on $\partial(D)$, say α, and hence the current distribution verifies $\sigma = \frac{1}{\alpha}\mu^*$ and $R = \frac{\rho}{2\pi}\alpha.$

Therefore to find the current distribution and the grid resistance it would suffice to solve problem (2). The extremal charges method consists on discretizing this problem in order to transform it into a sequence of linear programming problems whose solutions converge to the solution of problem (2). One of the keys of the discretization process, as make also the charge simulation method, is to distinguish between the points where the charge is placed and the points where the potential will be evaluated, as the charge simulation method do. Specifically, we consider a set of m charge points placed at the electrodes axes, with charge a_j in prescribed nodes p_j and their symmetrical p'_j, which corresponds to discretize $M^1(D)$; and we consider a set of n fixed evaluation points, x_i, placed on the electrodes boundary, which corresponds to the discretization of the grounding grid boundary. Figure 1 displays the discretization process of an electrode and its symmetrical.

Now the potential at an evaluation node x_i due to the m selected charges can be written as:

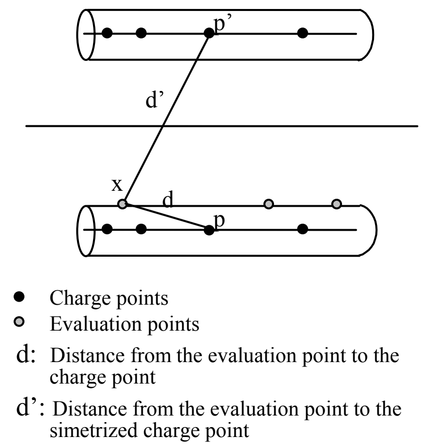

Fig. 1. Illustration of an electrode discretization.

$$V_i(a_1,\ldots,a_m) = \frac{\rho}{4\pi}\sum_{j=1}^{m} a_j\left(\frac{1}{d_{ij}} + \frac{1}{d'_{ij}}\right),$$

where $d_{ij} = \|x_i - p_j\|$ and $d'_{ij} = \|x_i - p'_j\|$. Then problem (2) is approximated by the following problem:

$$\min_{\substack{a_j \ge 0 \\ \sum_{j=1}^{m} a_j = 1}} \max_{i=1,\ldots,n} V_i(a_1,\ldots,a_m) \qquad (3)$$

Note that the unknowns of this optimization problem are the charges and the potentials V_i depending linearly on them. So if we introduce an additional parameter u, problem (3) can be rewritten as a linear programming problem:

$$\min\left\{u : 0 \le a_j \le 1, \sum_j a_j = 1, V_i - u \le 0\right\} \qquad (4)$$

The knowledge of the optimal solution $\left(a_1^*,\ldots,a_m^*,u^*\right)$ allows to estimate the current density as well as to know the grid resistance, the potential on the grid and on earth, and consequently the touch and step voltages. Specifically, u^* is an approximation of value α and hence $\frac{\rho}{2\pi}u^*$ is an approximation of the grid resistance R; $\left(a_1^*,\ldots,a_m^*\right)$ is an approximation of the current distribution and so the electrical potential at any point is approximated by:

$$V(x) = \frac{\rho}{4\pi} I \sum_{j=1}^{m} a_j^*\left(\frac{1}{\|x-p_j\|} + \frac{1}{\|x-p'_j\|}\right),$$

where I denotes the fault current.

In this way, we obtain the charge distribution as the solution of an optimization problem that, in particular, has the property of giving a positive solution which is necessary for being an approximation of the current density, [9],[11]. This optimization problem represents an alternative to the resolution of the linear system that other methods raise. In addition, the extremal charges method avoids the troubles associated with the calculus of the auto-influence coefficients that arise in other methods, since we distinguish between charge points and evaluation points, and so all the coefficients are upper bounded by the inverse of the electrode radius. This fact eliminates the possible numerical instability and at the same time keeps the convergence of the approximated solution. Clearly, the grounding grids usually have not extremities and when rods are incorporated to them, their extremities are far enough from the earth so that a rough approximation of the charge in the extremities does not produce instabilities in the computations. This does not eliminate the fact that the electrical potential is singular and it is divergent on one-dimensional elements. Therefore, if the segmentation of the electrodes is refined enough, instabilities will appear. Some of the authors computed the current density of a 2 meters long electrode by means of both the boundary element method making a segmentation of 25 elements with parabolic interpolation and the average potential method with 100 segments. In both cases, important fluctuations of the current density near the extremities were obtained being these fluctuations catastrophic when the segmentation was refined. These computations can be found in [8].

Besides, the independence between the number of charges and evaluation points enables us to discretize the grid by using different number of charges and evaluation points depending on the required accuracy. For instance, a decrease of the number of evaluation points leads to a linear programming problem in which the number of restrictions has decreased and therefore it is faster. On the other hand, a sophisticated geometry with plentiful electrodes will require to use enough charge points to get good approximations.

In order to get enough charge points without increase the number of variables, we broken up into segments the electrodes and we consider in each segment a charge of value a_j uniformly distributed in a few number of points. In this way we increase the number of charge points while keeping the number of unknowns, what only produces a little increase in the computation time of the coefficients of the constraint matrix. Moreover, from the algorithmic point of view, the characteristics of a linear programming problem allow to solve very quickly problems with a great number of constraints. Lastly, the fact of working in the extremal charges method with charge and evaluation points instead of electrode segments, as other methods do, releases grounding grids from geometrical constraints.

III. Description of the Computer Program

The computer program that solves problem (4) has been developed by the authors using FORTRAN code. The program works with an initial file loaded with all the parameters that describe the grounding system, such as, the number of electrodes, the number of ground rods (if there exist), the diameter for each electrode, the diameter for each ground rod, the number of evaluation points, the number of segments into which broken up the electrodes and the rods, the number of charges into each segment and the evaluation area of the ground surface. After reading the date file the program computes the constraint matrix. Finally, the program calls to routine E04MBF of the NAG15 library, that solves linear programming problems. Once the optimum value of the charges for each segment has been computed, it suffices to program the above-mentioned formulas to calculate the fundamental grid parameters.

It should be also noted that the computer time and the memory requirements are considerably modest. VAX 8600 has been used and CPU time required was about 50 seconds for a 16 mesh grid, 176 seconds for a 64 mesh grid and 184 seconds for a 256 mesh grid. This includes reading the geometrical data, computing the constraint matrix, solving the LP problem, getting the grid parameters and printing them out.

IV. Graphical Analysis of Some Grounding Grids

To show the applicability of the extremal charges method, we present the analysis of several academic grounding system configurations, for which the electrical potential and the fundamental parameters have been computed. The general characteristics of all the examples are the following ones:

- Homogeneous soil with resistivity of $100\Omega m$.
- Squared and symmetrical grids of $30m$ of side, buried at $50cm$.
- Electrodes of $2cm$ diameter.
- Rods $2m$ long with diameter of $3cm$.
- Fault current intensity of $1kA$.

At first place and in order to check the effectiveness of the extremal charges method we have made an exhaustive evaluation of the potential, due to the optimal charge, in the tangent plane to an equally spaced 64 mesh grid without rods. The result is displayed in Figure 2.

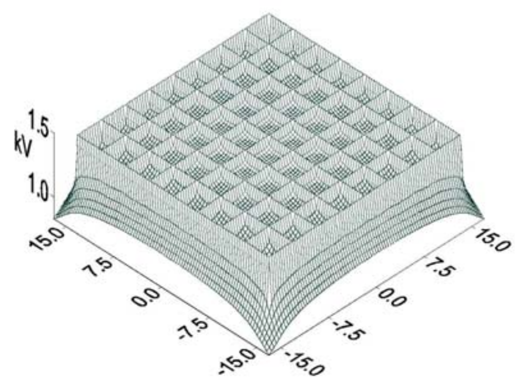

Fig. 2. Potential in the tangent plane to an equally spaced 64 mesh grid without rods

One can observe that the potential takes a unique noticeable value on the grid electrodes, whereas an important potential fall is produced in the meshes, being this extreme in the grid periphery. This behaviour becomes smoother when we space out from the grid, and in particular when we are placed on earth. However, the potential at the earth surface still depends on the grid shape, as we can see in Figure 3, where it is shown the potential in a square of $34x34\,m^2$ over the grid.

In Figure 4 we present the level curves of the potential on the earth surface to know something more about its behaviour. In the central zone the potential has slight variations, which agrees with the fact that the grid has enough number of electrodes; whereas in the periphery and mainly in the vertices high potential falls are noticeable.

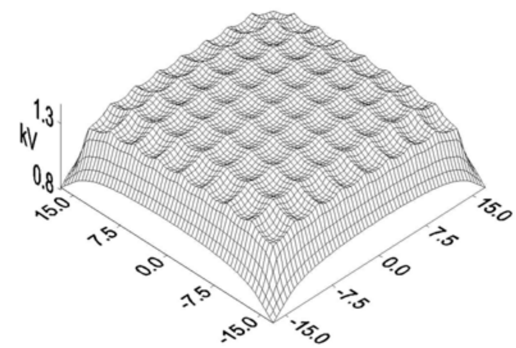

Fig. 3. Potential on the earth surface over an equally spaced 64 mesh grid without rods.

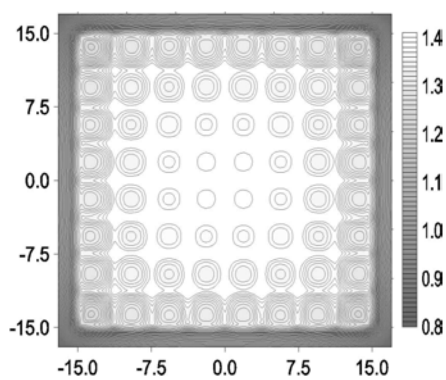

Fig. 4. Level curves of the potential on the earth surface over an equally spaced 64 mesh grid without rods.

For this example the value of the grid resistance is 1.482Ω; the touch voltage, computed as the maximum difference between the grounding grid potential and the potential on the earth surface of a $31.4\text{x}31.4m^2$ square that covers the grounding grid, is $0.488kV$ and the step voltage, computed as the maximum difference between potentials at points of the surface earth at a distance of one meter, is $0.212kV$. In the above described example, we have broken up into 450 segments one of the eight parts of the grid created by the symmetry, so that we are considering 450 different charge values. The linear programming algorithm computes the optimal charge values to produce constant potential on 3,780 points on the grid surface. The potential on the tangent plane to the grid due to this charge distribution has been calculated on 8,281 points, whereas the surface potential has been calculated on 3,721 points.

A. Improvement of the grid features

Because of shielding and fringing effects, that are produced in equally spaced grids, more current emanates from its peripheral electrodes, resulting in touch and step voltages on the corners of the grid higher than those in the center. To overcome these drawbacks, the technique more commonly employed is to place ground rods in the periphery of the grid and specially in the vertices. Also we can consider non equally spaced grids, see for instance [12]. Both techniques can improve the security with respect to touch and step voltage values.

Following these ideas and to show the versatility of the extremal charges method, we have moved progressively the grid electrodes from the center of the grid to its periphery keeping the symmetry properties, the electrode lengths and the number of meshes. For each one of the new performances we have evaluated the fundamental parameters. The results show an improvement of these values when considering non uniform geometries, which are more in keeping with the current distribution in squared grounding grids. Logically, the minimum value of all parameters does not occur for the same performance, so by optimal configuration we mean the one in which a better commitment between the parameters is achieved. We show the effect that the variation of the grid geometry produces by means of the analysis of a 16 mesh grid with one rod in each node of its periphery.

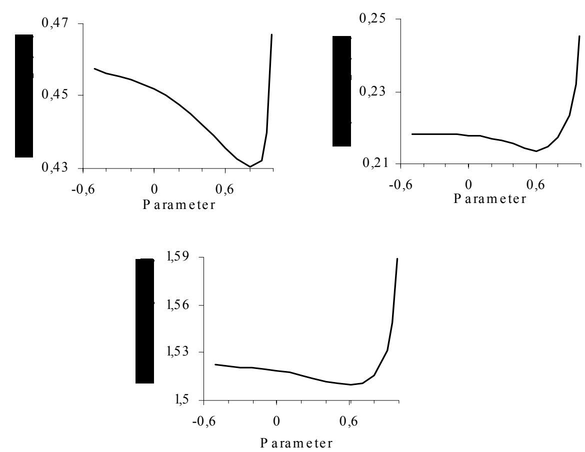

Fig. 5. Evolution of the fundamental parameters for a 16 mesh grid.

Figure 5 displays the evolution of the grid resistance and the touch and step voltage values for a 16 mesh grid according to different values of the *uniformity parameter*, $r \in [-1,1]$. The value zero of this parameter correspond to an equally spaced grid. The negative values correspond to move the grid electrodes toward the grid center and the positive values correspond to move the grid electrodes toward the grid periphery. The value $r = 1$ corresponds to the degenerate case in which the peripheral meshes vanish. As shown in Figure 5, the touch voltage attains its minimum at a value of the uniformity parameter too close to one, so that the distance between rods does not fulfil the standards about minimum distance between grid electrodes. Therefore, in this case the grid resistance and the step voltage values will determine the optimal configuration that is obtained for $r = 0.6$. This geometry can be guessed in Figure 7.

In Figures 6 and 7 we present the level curves of the potential on the earth surface for an equally spaced 16 mesh grid with rods and its optimized respectively. At the sight of the graphics we can verify that in the optimized grid there is less area of high potential. Moreover, the potential gradient in central meshes has increased but not enough as to exceed the maximum value of potential gradient, that is still achieved in the corners. The behaviour of the potential on the earth surface due to the optimized grid can be seen in Figure 8.

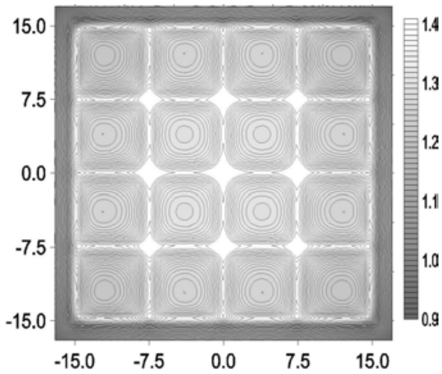

Fig. 6. Level curves of the potential on the earth surface over an equally spaced 16 mesh grid with rods.

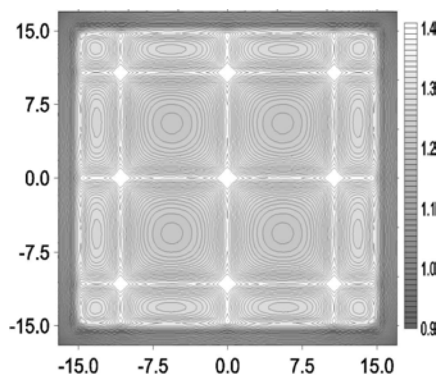

Fig. 7. Level curves of the potential on the earth surface over an optimized 16 mesh grid with rods.

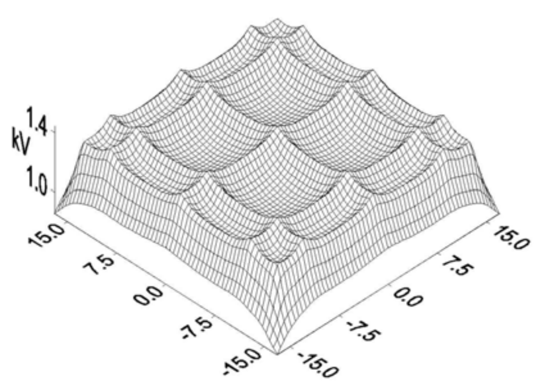

Fig. 8. Potential on the earth surface over an optimized 16 mesh grid with rods.

Lastly, we have computed the fundamental parameters of different grids and their optimized by using the extremal charges method. All of them verify the characteristics described in Section IV. The results are presented in Table I, where the resistance values are given in Ω and the voltage values in kV. Note that the greatest reduction in the values of the fundamental parameters is obtained when we add rods to the grids. However, the optimization of the grids also decreases the grid parameter values and as it is independent of the use of rods, it can be used as another tool in the grounding grid design.

TABLE I
FUNDAMENTAL PARAMETERS COMPARISON

16 mesh grid	without rods	vertex rods	node rods
R	1.586	1.555	1.516
opt. R	1.576	1.548	1.509
Vt	0.567	0.479	0.454
opt. Vt	0.549	0.466	0.436
Vs	0.239	0.228	0.217
opt. Vs	0.236	0.225	0.213
64 mesh grid	without rods	vertex rods	node rods
R	1.483	1.462	1.405
opt. R	1.47	1.451	1.398
Vt	0.488	0.417	0.376
opt. Vt	0.462	0.399	0.353
Vs	0.212	0.203	0.186
opt. Vs	0.214	0.204	0.185

The uniformity parameter is 0.6 in the three 16 mesh grid configurations and this triple coincidence suggests that it is the best performance. In the case of 64 mesh grid without rods or with rods in the vertices the value of the uniformity parameter is 0.4 where the smallest electrode is $1.5m$ long, whereas for the 64 mesh grid with rods on all peripheral nodes $r = 0.3$, that is, the smallest distance between rods is $2.1m$. We must observe that although the grid optimization does not produce considerable improvements in the grid parameters, these are achieved with the same amount of material. Well now, to obtain an improvement of the same magnitude, keeping the uniform structure of the grids, we could increase the electrode radius which would suppose a considerable increase of material. For instance, to obtain the value of the equivalence resistance of an optimized grid of 16 meshes, keeping the meshes uniform, we must use electrodes with a radius of $1.2cm$ which is an increase of 30% of material. If we make the same computation for a grid of 64 meshes we need electrodes of $1.4cm$ which is an increase of 50% of material. These savings of material are similar to the ones obtained in [12].

An analysis of the results showed in Table I suggests that the presence of rods in the vertices reduced the touch voltage more effectively than considering the geometry optimization. Well now, when a uniform grid contains enough electrodes, it can be more operative optimizing its design than increasing the electrode number. For instance, doubling the number of electrodes, in the 64 mesh grid, provides a 3.6% of reduction in the equivalence resistance. However, if we add rods in the periphery of the grid, which will only suppose and increase of 26% of material, we obtain a 5.3% of reduction in the equivalence resistance and a 5.7% of reduction if, in addition, we consider grid optimization.

V. CONCLUSIONS

The grid design in high voltage substations requires a straightforward, versatile and accurate method to compute the electrical potential. These properties can be achieved by the average potential methods whenever a good electrode segmentation is made and the system of equations that determines the current density is resolved.

In this paper we present a new methodology , that allows to obtain the current density by solving a linear programming problem. We have applied this method, called extremal charges method, to different electrode grids and we have verified its accuracy. On the other hand the distinction between charge and evaluation points allows us to adapt the number of each one to the different phases of the design.

The method versatility has allowed us to tackle the analysis of the influence of the grid geometry in the potential and definitely in the fundamental parameters. This enables to obtain, after optimization, simple configurations in which the fundamental parameter values are improved keeping fixed the quantity of grid material. Besides, we have shown that the optimization process can produce a material saving while the fundamental parameter values are in the limits of a safely tolerable voltage.

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VII. BIOGRAPHIES

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